Close Wed: HW_3A,3B,3C
(complete sooner!)
Exam 1 is Thurs (4.9, 5.1-5.5, 6.1-6.3)

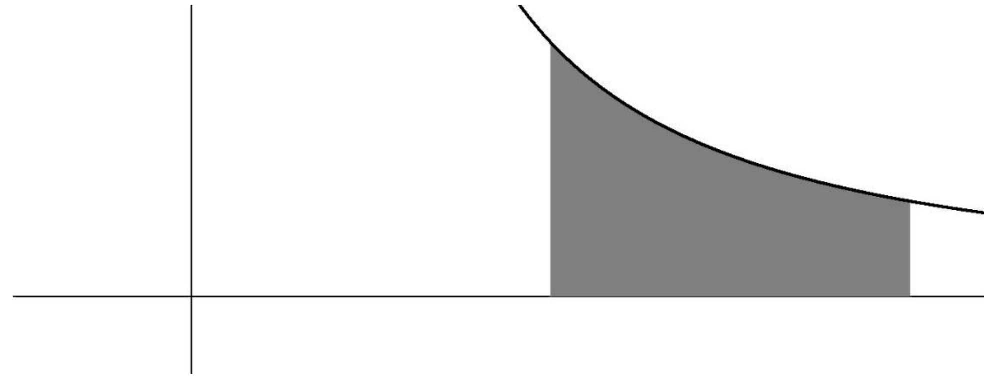
Entry Task:
Consider the region *R* bounded by
$y = x^3$, $y = 8$, and $x = 0$.
Set up the integrals that would give the volume of the solid obtained by rotating *R* about the ….

(a) … *x*-axis.
(b) … *y*-axis.
(c) … vertical line *x* = -10.

Example:

Let R be the region bounded by

$$y = \frac{1}{x^2} + \frac{1}{x}, y = 0, x = 1, x = 2.$$



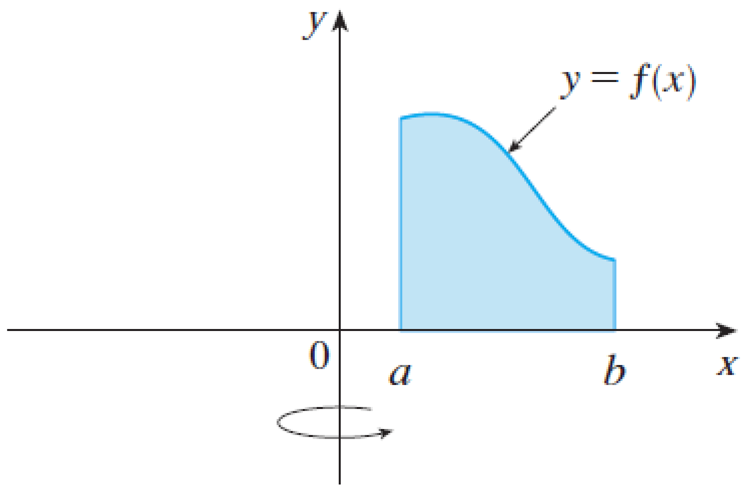
Consider the solid obtained by rotating about the **y-axis**.

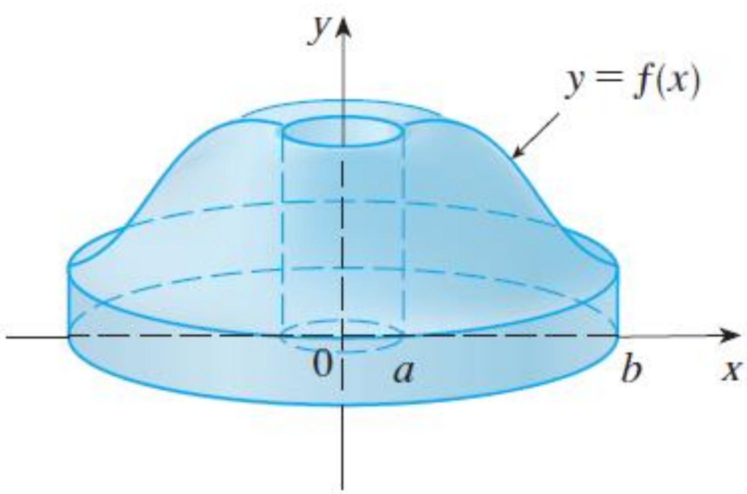
Try to use cross-sectional slicing… why is this difficult/messy?

6.3 Volumes Using Cylindrical Shells

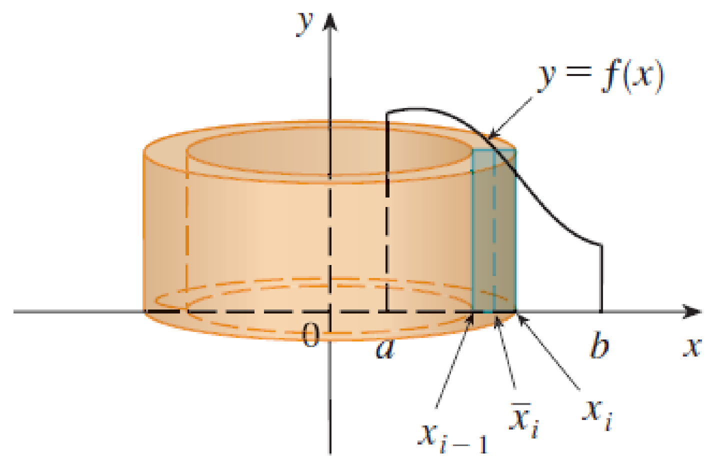
Visual Motivation:

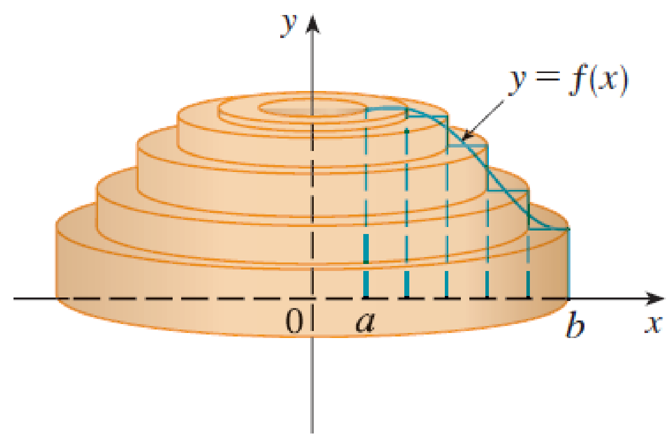
Consider the solid





We want to use "dx", but that breaks the region into thin vertical subdivisions and rotating those gives a new shape, "cylindrical shells"





Derivation:

The pattern for the volume of one thin cylindrical shell is

VOLUME ≈ (surface area)(thickness)

= $SA(x_i)\ \Delta x$

= 2π(radius)(height)(thickness)

Thus, if we can find a formula, $SA(x_i)$, for the surface area of a typical cylindrical shell, then

Thin Shell Volume ≈ $SA(x_i)\ \Delta x$,

$$\text{Total Volume} \approx \sum_{i=1}^{n} \mathrm{SA}(\mathrm{x}_i)\ \Delta \mathrm{x}$$

$$\text{Exact Volume} = \lim_{n\to\infty} \sum_{i=1}^{n} \mathrm{SA}(\mathrm{x}_i)\Delta \mathrm{x}$$

$$\text{Volume} = \int_a^b SA(x)dx$$

$$= \int_a^b 2\pi(radius)(height)dx$$

Volume using cylindrical shells

1. Draw region. Cut **parallel** to rotation axis. Label *x* if that cut crosses the *x*-axis (and *y* if *y*-axis). Label **everything** in terms this variable.

2. Formula for surface area of cylindrical shell?

   SA = (Circumference)(Height)

   = 2π(Radius)(Height)

3. Integrate the SA formula.

Example:

Let R be the region bounded by

$$y = \frac{1}{x^2} + \frac{1}{x}, y = 0, x = 1, x = 2.$$

Set up an integral for the volume obtained by rotating R about the ***y*-axis**.

Example: Let R be the region in the first quadrant that is bounded by $x = \sqrt{y+1}$ and $y = 1$.
Find the volume obtained by rotating R about the ***x*-axis**.

<u>Flow chart of all Volume of Revolution Problems</u>

***Step 0*: Draw an accurate picture!!! (Always draw a picture)**

Step 1: Choose and label the variable (based on the region and given equations)
If *x*, draw a typical **vertical** thin approximating rectangle at *x*.
If *y*, draw a typical **horizontal** thin approximating rectangle at *y*.

Step 2: Is the approximating rectangle perpendicular or parallel to the rotation axis?

Perpendicular → *Cross-sections*:

$$\text{Volume} = \int_a^b \left(\pi(\text{outer})^2 - \pi(\text{inner})^2\right)(\text{dx or dy})$$

Parallel → *Shells*:

$$\text{Volume} = \int_a^b 2\pi(\text{radius})(\text{height})(\text{dx or dy})$$

Step 3: Write everything in terms of the desired variable and fill in patterns.
Then integrate.

The above method is how you should approach problems, but if you are still having trouble seeing which variable goes with which method here is a summary:

Axis of rotation	**Disc/Washer**	**Shells**
x-axis (or any horizontal axis)	dx	dy
y-axis (or any vertical axis)	dy	dx

Example:

Let R be the region bounded by $y = x^3$, $y = 4x$ between $\mathrm{x} = 1$ and $x = 2$.

1. Set up an integral for the volume of the solid obtained by rotating R **about the y-axis**.

2. Set up an integral for the volume of the solid obtained by rotating R **about the x-axis**.

3. Set up an integral for the volume of the solid obtained by rotating R **about the vertical line x=3**.